

An application of interval-valued neutrosophic soft graphs in a decision making problem

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Abstract: In this paper, we introduce the concept of interval-valued neutrosophic soft graphs and gave some new operations such as parametric \wedge -intersection and parametric \vee -union on interval-valued neutrosophic soft graphs. We have also applied the concept of interval-valued neutrosophic soft graph in a decision making problem and then gave an algorithm for the selection of optimal object.

Keywords: Interval-valued neutrosophic soft sets, interval-valued neutrosophic soft graphs, decision making problems.

1 Introduction

The concept of neutrosophic set which is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information was firstly discussed by Smarandache [1]. The neutrosophic sets are characterized by a truth-membership function T , an indeterminacy-membership function I and a falsity membership function F independently, which are within the real standard or nonstandard unit interval $] -0, 1+[$. Wang et al. [2] introduced the concept of interval-valued neutrosophic sets. Some works on neutrosophic sets, interval valued neutrosophic sets and their applications may be found on [3,4,5,6,7,8].

The concept of soft set theory was initiated by Molodtsov [9]. It has been showed that soft sets have potential applications in different fields. The definition of neutrosophic soft sets was firstly given by Maji [10]. He also discussed many operations such as union, intersection and complement on neutrosophic soft sets. The concept of interval-valued neutrosophic soft set which is a generalization of neutrosophic soft sets was given by Deli [11].

Graph theory was firstly introduced by Euler [12]. Since then graph theory has become the most important part of combinatorial mathematics. A graph is used to create a relationship between a given set of elements. Each element can be represented by a vertex and the relationship between them can be represented by an edge. The concept of graph has been applied to different algebraic structures in mathematics.

In this paper, we define some new operations such as parametric \wedge -intersection and parametric \vee -union on interval-valued neutrosophic soft graphs. We also present an application of interval-valued neutrosophic soft graph in a decision making problem and give an algorithm for the selection of optimal object based on given sets of information.

2 Preliminaries

Definition 1. [1] A neutrosophic set A on the universe of discourse U is defined as $A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in U \}$, where the functions $T, I, F : U \rightarrow [0, 1]$ define a truth-membership function, an indeterminacy-membership

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function and a falsity-membership function of an element $x \in U$ for a set A , respectively, with the condition $-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. The family of all neutrosophic sets on U is denoted by $\mathcal{N}(U)$.

Definition 2. [10] Let U be an initial universe set, E be a set of parameters and $A \subseteq E$. If F is a mapping given by $F : A \rightarrow \mathcal{N}(U)$, then a pair (F, A) is called a neutrosophic soft set over U .

Definition 3. [11] Let X be a space of objects with a generic element in X denoted by x . An interval-valued neutrosophic set (IVN-sets) A in X is characterized by truth-membership function T_A , an indeterminacy-membership function I_A and a falsity-membership function F_A . For each point $x \in X$, T_A , I_A and $F_A \subseteq [0, 1]$. For this reason, we can represent each membership function for $x \in X$ as follows:

$$T_A(x) = [\inf T_A(x), \sup T_A(x)], I_A(x) = [\inf I_A(x), \sup I_A(x)] \text{ and } F_A(x) = [\inf F_A(x), \sup F_A(x)]$$

Thus, an IVN-set over X can be represented by

$$A = \{ \langle x, [\inf T_A(x), \sup T_A(x)], [\inf I_A(x), \sup I_A(x)], [\inf F_A(x), \sup F_A(x)] \rangle \mid x \in X \}$$

Definition 4. [11] Let X be an initial universe set, $IVN(X)$ denotes the set of all interval-valued neutrosophic sets of X and E be a set of parameters that describe the elements of X . An interval valued neutrosophic soft set (Y, E) over X is a set defined by a set valued function Y representing a mapping $Y : E \rightarrow IVN(X)$. It can be written a set of ordered pairs $(Y, E) = \{ \langle e, Y(e) \rangle : e \in E \}$. Here Y is called approximate functions of the (Y, E) and $Y(e)$ is called e -approximate value of $e \in E$.

We denote the set of all interval-valued neutrosophic soft sets over X by $IVNS(X)$.

Definition 5. [12] A graph G^* consists of set of finite objects $V = \{v_1, v_2, v_3, \dots, v_n\}$ called vertices and other set $E = \{e_1, e_2, e_3, \dots, e_n\}$ whose elements are called edges. Usually a graph is denoted as $G^* = (V, E)$.

Definition 6. [13] An interval-valued neutrosophic graph of a graph $G^* = (V, E)$ is given by a pair $G = (A, B)$, where $A = \langle [T_A^-, T_A^+], [I_A^-, I_A^+], [F_A^-, F_A^+] \rangle$ is an interval-valued neutrosophic set on V and $B = \langle [T_B^-, T_B^+], [I_B^-, I_B^+], [F_B^-, F_B^+] \rangle$ is an interval-valued neutrosophic relation on E such that

$$\begin{aligned} T_B^-(v_i, v_j) &\leq \min\{T_A^-(v_i), T_A^-(v_j)\} & T_B^+(v_i, v_j) &\leq \min\{T_A^+(v_i), T_A^+(v_j)\} \\ I_B^-(v_i, v_j) &\geq \max\{I_A^-(v_i), I_A^-(v_j)\} & I_B^+(v_i, v_j) &\geq \max\{I_A^+(v_i), I_A^+(v_j)\} \\ F_B^-(v_i, v_j) &\geq \max\{F_A^-(v_i), F_A^-(v_j)\} & F_B^+(v_i, v_j) &\geq \max\{F_A^+(v_i), F_A^+(v_j)\} \end{aligned}$$

for all $(v_i, v_j) \in E$.

Definition 7. [14] An interval-valued neutrosophic soft graph $\tilde{G} = (G^*, K, M, A)$ is a 4-tuple such that

- (i) $G^* = (V, E)$ is a simple graph,
- (ii) A is a non-empty set of parameters,
- (iii) (K, A) is an interval-valued neutrosophic soft set over V ,
- (iv) (M, A) is an interval-valued neutrosophic soft set over E ,
- (v) $(K(e), M(e))$ is an interval-valued neutrosophic soft subgraph of G^* for all $e \in A$. That is,

$$\begin{aligned} \inf T_{M(e)}(xy) &\leq \min\{\inf T_{K(e)}(x), \inf T_{K(e)}(y)\} & \sup T_{M(e)}(xy) &\leq \min\{\sup T_{K(e)}(x), \sup T_{K(e)}(y)\} \\ \inf I_{M(e)}(xy) &\geq \max\{\inf I_{K(e)}(x), \inf I_{K(e)}(y)\} & \sup I_{M(e)}(xy) &\geq \max\{\sup I_{K(e)}(x), \sup I_{K(e)}(y)\} \\ \inf F_{M(e)}(xy) &\geq \max\{\inf F_{K(e)}(x), \inf F_{K(e)}(y)\} & \sup F_{M(e)}(xy) &\geq \max\{\sup F_{K(e)}(x), \sup F_{K(e)}(y)\} \end{aligned}$$

The interval-valued neutrosophic graph $(K(e), M(e))$ is denoted by $H(e)$ for convenience. An interval-valued neutrosophic soft graph is a parametrized family of interval-valued neutrosophic graphs. The set of all interval-valued neutrosophic soft graphs of G^* is denoted by $IVN(G^*)$.

Example 1. Consider a simple graph $G^* = (V, E)$ such that $V = \{v_1, v_2, v_3\}$ and $E = \{v_1v_2, v_2v_3, v_1v_3\}$. Let $A = \{e_1, e_2\}$ be set of parameter and let (K, A) be an interval-valued neutrosophic soft set over V with its approximate function $K : A \rightarrow IVN(V)$ defined by

$$K(e_1) = \{ \langle v_1, [0.5, 0.6], [0.3, 0.4], [0.2, 0.3] \rangle, \langle v_2, [0.4, 0.5], [0.3, 0.4], [0.6, 0.7] \rangle, \langle v_3, [0.6, 0.7], [0.2, 0.3], [0.1, 0.2] \rangle \},$$

$$K(e_2) = \{ \langle v_1, [0.3, 0.5], [0.7, 0.8], [0.4, 0.5] \rangle, \langle v_2, [0.2, 0.3], [0.4, 0.5], [0.4, 0.5] \rangle, \langle v_3, [0.2, 0.4], [0.3, 0.5], [0.1, 0.2] \rangle \}.$$

Let (M, A) be an interval-valued neutrosophic soft set over E with its approximate function $M : E \rightarrow IVN(E)$ defined by

$$M(e_1) = \{ \langle v_1v_2, [0.3, 0.4], [0.4, 0.5], [0.7, 0.8] \rangle, \langle v_2v_3, [0.1, 0.2], [0.5, 0.6], [0.6, 0.8] \rangle, \langle v_3v_1, [0.2, 0.3], [0.3, 0.6], [0.4, 0.5] \rangle \},$$

$$M(e_2) = \{ \langle v_1v_2, [0.2, 0.3], [0.7, 0.8], [0.5, 0.6] \rangle, \langle v_2v_3, [0.1, 0.2], [0.6, 0.7], [0.5, 0.6] \rangle, \langle v_3v_1, [0.1, 0.4], [0.8, 0.9], [0.6, 0.7] \rangle \}.$$

Clearly, $H(e_1) = (K(e_1), M(e_1))$ and $H(e_2) = (K(e_2), M(e_2))$ are interval-valued neutrosophic graphs corresponding to the paramteters e_1 and e_2 as shown in figure below, respectively.

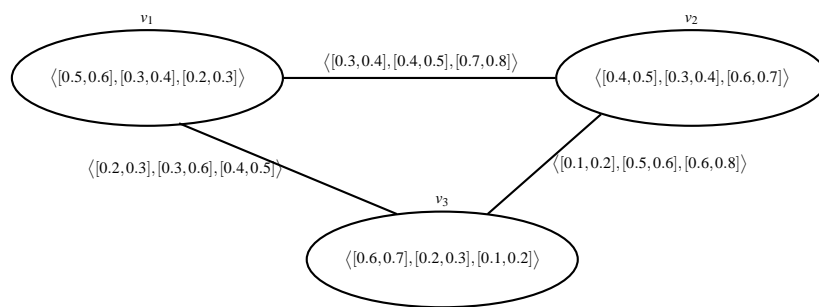


Fig. 1: $H(e_1)$ interval valued neutrosophic graph

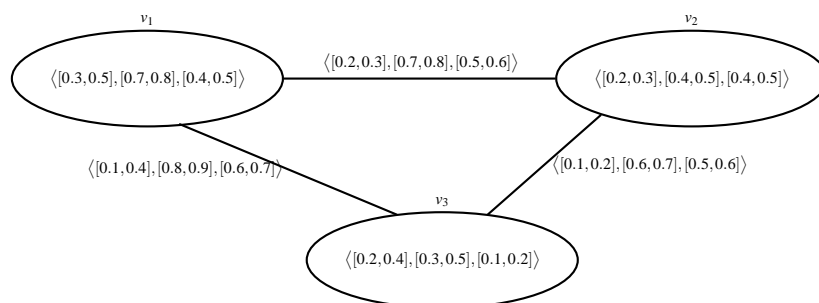


Fig. 2: $H(e_2)$ interval valued neutrosophic graph

Hence $\tilde{G} = \{H(e_1), H(e_2)\}$ is an interval-valued neutrosophic soft graph of G^* . Tabular representation of an interval-valued neutrosophic soft graph \tilde{G} is given in Table 1.

Table 1: Interval-valued neutrosophic soft graph \tilde{G}

K	v_1	v_2	v_3
e_1	$\langle [0.5, 0.6], [0.3, 0.4], [0.2, 0.3] \rangle$	$\langle [0.4, 0.5], [0.3, 0.4], [0.6, 0.7] \rangle$	$\langle [0.6, 0.7], [0.2, 0.3], [0.1, 0.2] \rangle$
e_2	$\langle [0.3, 0.5], [0.7, 0.8], [0.4, 0.5] \rangle$	$\langle [0.2, 0.3], [0.4, 0.5], [0.4, 0.5] \rangle$	$\langle [0.2, 0.4], [0.3, 0.5], [0.1, 0.2] \rangle$
M	$v_1 v_2$	$v_2 v_3$	$v_3 v_1$
e_1	$\langle [0.3, 0.4], [0.4, 0.5], [0.7, 0.8] \rangle$	$\langle [0.1, 0.2], [0.5, 0.6], [0.6, 0.8] \rangle$	$\langle [0.2, 0.3], [0.3, 0.6], [0.4, 0.5] \rangle$
e_2	$\langle [0.2, 0.3], [0.7, 0.8], [0.5, 0.6] \rangle$	$\langle [0.1, 0.2], [0.6, 0.7], [0.5, 0.6] \rangle$	$\langle [0.1, 0.4], [0.8, 0.9], [0.6, 0.7] \rangle$

Definition 8. [14] Let $\tilde{G}_1 = (G^*, K_1, M_1, A)$ and $\tilde{G}_2 = (G^*, K_2, M_2, B)$ be two interval-valued neutrosophic soft graphs of $G^* = (V, E)$. Then \tilde{G}_1 is an interval valued neutrosophic soft subgraph of \tilde{G}_2 if

- (i) $A \subseteq B$
- (ii) $H_1(e)$ is a subgraph of $H_2(e)$ for all $e \in A$.

Definition 9. Let $\tilde{G} = (G^*, K, M, A)$ be an interval-valued neutrosophic soft graph of $G^* = (V, E)$ and $A = \{e_1, e_2, \dots, e_n\}$ be a set of parameters. The parametric \wedge -intersection of subgraphs of \tilde{G} is denoted by

$$M(e) = M(e_1) \tilde{\wedge} M(e_2) \tilde{\wedge} \dots \tilde{\wedge} M(e_n)$$

where $e = e_1 \wedge e_2 \wedge \dots \wedge e_n$ and for all $xy \in E$

$$\begin{aligned} \inf T_{M(e)}(xy) &= \min\{\inf T_{M(e_1)}(xy), \inf T_{M(e_2)}(xy), \dots, \inf T_{M(e_n)}(xy)\} \\ \sup T_{M(e)}(xy) &= \min\{\sup T_{M(e_1)}(xy), \sup T_{M(e_2)}(xy), \dots, \sup T_{M(e_n)}(xy)\} \\ \inf I_{M(e)}(xy) &= \max\{\inf I_{M(e_1)}(xy), \inf I_{M(e_2)}(xy), \dots, \inf I_{M(e_n)}(xy)\} \\ \sup I_{M(e)}(xy) &= \max\{\sup I_{M(e_1)}(xy), \sup I_{M(e_2)}(xy), \dots, \sup I_{M(e_n)}(xy)\} \\ \inf F_{M(e)}(xy) &= \max\{\inf F_{M(e_1)}(xy), \inf F_{M(e_2)}(xy), \dots, \inf F_{M(e_n)}(xy)\} \\ \sup F_{M(e)}(xy) &= \max\{\sup F_{M(e_1)}(xy), \sup F_{M(e_2)}(xy), \dots, \sup F_{M(e_n)}(xy)\}. \end{aligned}$$

Definition 10. Let $\tilde{G} = (G^*, K, M, A)$ be an interval-valued neutrosophic soft graph of $G^* = (V, E)$ and $A = \{e_1, e_2, \dots, e_n\}$ be a set of parameters. The parametric \vee -union of subgraphs of \tilde{G} is denoted by

$$M(e) = M(e_1) \tilde{\vee} M(e_2) \tilde{\vee} \dots \tilde{\vee} M(e_n)$$

where $e = e_1 \vee e_2 \vee \dots \vee e_n$ and for all $xy \in E$

$$\begin{aligned} \inf T_{M(e)}(xy) &= \min\{\inf T_{M(e_1)}(xy), \inf T_{M(e_2)}(xy), \dots, \inf T_{M(e_n)}(xy)\} \\ \sup T_{M(e)}(xy) &= \min\{\sup T_{M(e_1)}(xy), \sup T_{M(e_2)}(xy), \dots, \sup T_{M(e_n)}(xy)\} \\ \inf I_{M(e)}(xy) &= \max\{\inf I_{M(e_1)}(xy), \inf I_{M(e_2)}(xy), \dots, \inf I_{M(e_n)}(xy)\} \\ \sup I_{M(e)}(xy) &= \max\{\sup I_{M(e_1)}(xy), \sup I_{M(e_2)}(xy), \dots, \sup I_{M(e_n)}(xy)\} \\ \inf F_{M(e)}(xy) &= \max\{\inf F_{M(e_1)}(xy), \inf F_{M(e_2)}(xy), \dots, \inf F_{M(e_n)}(xy)\} \\ \sup F_{M(e)}(xy) &= \max\{\sup F_{M(e_1)}(xy), \sup F_{M(e_2)}(xy), \dots, \sup F_{M(e_n)}(xy)\}. \end{aligned}$$

Example 2. Consider a simple graph $G^* = (V, E)$ such that

$$V = \{v_1, v_2, v_3, v_4, v_5\} \text{ and } E = \{v_1 v_2, v_1 v_3, v_1 v_4, v_1 v_5, v_2 v_3, v_2 v_4, v_2 v_5, v_3 v_4, v_3 v_5, v_4 v_5\}.$$

Let $A = \{e_1, e_2, e_3\}$ be a set of parameters and consider interval-valued neutrosophic soft graph $\tilde{G} = (G^*, K, M, A)$ as follows.

$$\tilde{G} = \{H(e_1), H(e_2), H(e_3)\} = \{(K(e_1), M(e_1)), (K(e_2), M(e_2)), (K(e_3), M(e_3))\}$$

$$\begin{aligned} K(e_1) = & \{ \langle v_1, [0.2, 0.4], [0.3, 0.5], [0.6, 0.8] \rangle, \langle v_2, [0.1, 0.3], [0.4, 0.7], [0.3, 0.6] \rangle, \\ & \langle v_3, [0.3, 0.5], [0.2, 0.3], [0.4, 0.5] \rangle, \langle v_4, [0.5, 0.6], [0.5, 0.8], [0.2, 0.4] \rangle, \\ & \langle v_5, [0.4, 0.7], [0.6, 0.9], [0.7, 0.9] \rangle \} \\ M(e_1) = & \{ \langle v_1 v_2, [0.1, 0.3], [0.6, 0.8], [0.7, 0.8] \rangle, \langle v_1 v_3, [0.2, 0.4], [0.5, 0.6], [0.8, 0.9] \rangle, \\ & \langle v_1 v_5, [0.1, 0.2], [0.8, 0.9], [0.7, 0.9] \rangle, \langle v_2 v_3, [0.1, 0.3], [0.5, 0.7], [0.4, 0.6] \rangle, \\ & \langle v_2 v_4, [0.1, 0.2], [0.6, 0.9], [0.5, 0.7] \rangle, \langle v_2 v_5, [0.1, 0.3], [0.8, 0.9], [0.7, 0.9] \rangle, \\ & \langle v_3 v_4, [0.3, 0.4], [0.7, 0.9], [0.8, 0.9] \rangle, \langle v_3 v_5, [0.2, 0.5], [0.8, 0.9], [0.7, 0.9] \rangle \}. \end{aligned}$$

$$\begin{aligned} K(e_2) = & \{ \langle v_1, [0.2, 0.3], [0.6, 0.7], [0.2, 0.3] \rangle, \langle v_2, [0.2, 0.4], [0.3, 0.5], [0.3, 0.4] \rangle, \\ & \langle v_3, [0.6, 0.8], [0.3, 0.4], [0.6, 0.7] \rangle, \langle v_4, [0.1, 0.2], [0.5, 0.6], [0.7, 0.8] \rangle, \\ & \langle v_5, [0.3, 0.4], [0.6, 0.7], [0.8, 0.9] \rangle \}, \\ M(e_2) = & \{ \langle v_1 v_2, [0.2, 0.3], [0.7, 0.8], [0.5, 0.7] \rangle, \langle v_1 v_4, [0.1, 0.2], [0.8, 0.9], [0.7, 0.9] \rangle, \\ & \langle v_1 v_5, [0.3, 0.4], [0.7, 0.9], [0.8, 0.9] \rangle, \langle v_2 v_3, [0.1, 0.3], [0.4, 0.5], [0.7, 0.8] \rangle, \\ & \langle v_2 v_5, [0.2, 0.4], [0.8, 0.9], [0.8, 0.9] \rangle, \langle v_3 v_4, [0.1, 0.2], [0.6, 0.7], [0.8, 0.9] \rangle \}. \end{aligned}$$

$$\begin{aligned} K(e_3) = & \{ \langle v_1, [0.3, 0.5], [0.4, 0.5], [0.5, 0.6] \rangle, \langle v_2, [0.2, 0.4], [0.3, 0.4], [0.6, 0.7] \rangle, \\ & \langle v_3, [0.1, 0.3], [0.5, 0.7], [0.4, 0.6] \rangle, \langle v_4, [0.4, 0.6], [0.2, 0.3], [0.3, 0.5] \rangle, \\ & \langle v_5, [0.3, 0.4], [0.6, 0.7], [0.7, 0.8] \rangle \} \\ M(e_3) = & \{ \langle v_1 v_2, [0.2, 0.3], [0.5, 0.6], [0.7, 0.8] \rangle, \langle v_1 v_5, [0.3, 0.4], [0.7, 0.8], [0.8, 0.9] \rangle, \\ & \langle v_2 v_3, [0.1, 0.2], [0.6, 0.7], [0.7, 0.9] \rangle, \langle v_2 v_5, [0.2, 0.4], [0.8, 0.9], [0.7, 0.8] \rangle, \\ & \langle v_3 v_4, [0.1, 0.3], [0.5, 0.8], [0.6, 0.7] \rangle, \langle v_3 v_5, [0.1, 0.2], [0.7, 0.8], [0.7, 0.9] \rangle, \\ & \langle v_4 v_5, [0.1, 0.4], [0.6, 0.7], [0.8, 0.9] \rangle \}. \end{aligned}$$

Clearly, $H(e_1)$, $H(e_2)$ and $H(e_3)$ are subgraphs of \tilde{G} . They are shown in matrix form in the below for e_1 , e_2 and e_3 respectively.

$$H(e_1) = \begin{bmatrix} \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.3], [0.6, 0.8], [0.7, 0.8] \rangle & \langle [0.2, 0.4], [0.5, 0.6], [0.8, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.8, 0.9], [0.7, 0.9] \rangle \\ \langle [0.1, 0.3], [0.6, 0.8], [0.7, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.3], [0.5, 0.7], [0.4, 0.6] \rangle & \langle [0.1, 0.2], [0.6, 0.9], [0.5, 0.7] \rangle & \langle [0.1, 0.3], [0.8, 0.9], [0.7, 0.9] \rangle \\ \langle [0.2, 0.4], [0.5, 0.6], [0.8, 0.9] \rangle & \langle [0.1, 0.3], [0.5, 0.7], [0.4, 0.6] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.3, 0.4], [0.7, 0.9], [0.8, 0.9] \rangle & \langle [0.2, 0.5], [0.8, 0.9], [0.7, 0.9] \rangle \\ \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.6, 0.9], [0.5, 0.7] \rangle & \langle [0.3, 0.4], [0.7, 0.9], [0.8, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \\ \langle [0.1, 0.2], [0.8, 0.9], [0.7, 0.9] \rangle & \langle [0.1, 0.3], [0.8, 0.9], [0.7, 0.9] \rangle & \langle [0.2, 0.5], [0.8, 0.9], [0.7, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \end{bmatrix}$$

$$H(e_2) = \begin{bmatrix} \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.2, 0.3], [0.7, 0.8], [0.5, 0.7] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.8, 0.9], [0.7, 0.9] \rangle & \langle [0.3, 0.4], [0.7, 0.9], [0.8, 0.9] \rangle \\ \langle [0.2, 0.3], [0.7, 0.8], [0.5, 0.7] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.3], [0.4, 0.5], [0.7, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.2, 0.4], [0.8, 0.9], [0.8, 0.9] \rangle \\ \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.3], [0.4, 0.5], [0.7, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.6, 0.7], [0.8, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \\ \langle [0.1, 0.2], [0.8, 0.9], [0.7, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.6, 0.7], [0.8, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \\ \langle [0.3, 0.4], [0.7, 0.9], [0.8, 0.9] \rangle & \langle [0.2, 0.4], [0.8, 0.9], [0.8, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \end{bmatrix}$$

$$H(e_3) = \begin{bmatrix} \langle [0,0], [0,0], [0,0] \rangle & \langle [0.2,0.3], [0.5,0.6], [0.7,0.8] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.3,0.4], [0.7,0.8], [0.8,0.9] \rangle \\ \langle [0.2,0.3], [0.5,0.6], [0.7,0.8] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.2], [0.6,0.7], [0.7,0.9] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.2,0.4], [0.8,0.9], [0.7,0.8] \rangle \\ \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.2], [0.6,0.7], [0.7,0.9] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.3], [0.5,0.8], [0.6,0.7] \rangle & \langle [0.1,0.2], [0.7,0.8], [0.7,0.9] \rangle \\ \langle [0,0], [0,0], [0,0] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.3], [0.5,0.8], [0.6,0.7] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.4], [0.6,0.7], [0.8,0.9] \rangle \\ \langle [0.3,0.4], [0.7,0.8], [0.8,0.9] \rangle & \langle [0.2,0.4], [0.8,0.9], [0.7,0.8] \rangle & \langle [0.1,0.2], [0.7,0.8], [0.7,0.9] \rangle & \langle [0.1,0.4], [0.6,0.7], [0.8,0.9] \rangle & \langle [0,0], [0,0], [0,0] \rangle \end{bmatrix}$$

By using Definition 9, we obtain the parametric \wedge —intersection of subgraphs of \tilde{G} as follows.

$$H_{\wedge}(e) = \begin{bmatrix} \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.3], [0.7,0.8], [0.7,0.8] \rangle & \langle [0,0], [0.5,0.6], [0.8,0.9] \rangle & \langle [0,0], [0.8,0.9], [0.7,0.9] \rangle & \langle [0.1,0.2], [0.8,0.9], [0.8,0.9] \rangle \\ \langle [0.1,0.3], [0.7,0.8], [0.7,0.8] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.2], [0.6,0.7], [0.7,0.9] \rangle & \langle [0,0], [0.6,0.9], [0.5,0.7] \rangle & \langle [0.1,0.3], [0.8,0.9], [0.8,0.9] \rangle \\ \langle [0,0], [0.5,0.6], [0.8,0.9] \rangle & \langle [0.1,0.2], [0.6,0.7], [0.7,0.9] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.2], [0.7,0.9], [0.8,0.9] \rangle & \langle [0,0], [0.8,0.9], [0.7,0.9] \rangle \\ \langle [0,0], [0.8,0.9], [0.7,0.9] \rangle & \langle [0,0], [0.7,0.8], [0.8,0.9] \rangle & \langle [0.1,0.2], [0.7,0.9], [0.8,0.9] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0,0], [0.6,0.7], [0.8,0.9] \rangle \\ \langle [0.1,0.2], [0.8,0.9], [0.8,0.9] \rangle & \langle [0.1,0.3], [0.8,0.9], [0.8,0.9] \rangle & \langle [0,0], [0.7,0.9], [0.8,0.9] \rangle & \langle [0,0], [0.6,0.7], [0.8,0.9] \rangle & \langle [0,0], [0,0], [0,0] \rangle \end{bmatrix}$$

By using Definition 10, we obtain the parametric \vee —union of subgraphs of \tilde{G} as follows.

$$H_{\vee}(e) = \begin{bmatrix} \langle [0,0], [0,0], [0,0] \rangle & \langle [0.2,0.3], [0.5,0.6], [0.7,0.8] \rangle & \langle [0.2,0.4], [0,0], [0,0] \rangle & \langle [0.1,0.2], [0,0], [0,0] \rangle & \langle [0.3,0.4], [0.7,0.8], [0.8,0.9] \rangle \\ \langle [0.2,0.3], [0.5,0.6], [0.7,0.8] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.3], [0.4,0.5], [0.4,0.6] \rangle & \langle [0.1,0.2], [0,0], [0,0] \rangle & \langle [0.2,0.4], [0.8,0.9], [0.7,0.9] \rangle \\ \langle [0.2,0.4], [0,0], [0,0] \rangle & \langle [0.1,0.3], [0.4,0.5], [0.4,0.6] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.3,0.4], [0.5,0.7], [0.6,0.7] \rangle & \langle [0.2,0.5], [0,0], [0,0] \rangle \\ \langle [0.1,0.2], [0,0], [0,0] \rangle & \langle [0.2,0.3], [0,0], [0,0] \rangle & \langle [0.3,0.4], [0.5,0.7], [0.6,0.7] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.4], [0,0], [0,0] \rangle \\ \langle [0.3,0.4], [0.7,0.8], [0.7,0.9] \rangle & \langle [0.2,0.4], [0.8,0.9], [0.7,0.8] \rangle & \langle [0.2,0.5], [0,0], [0,0] \rangle & \langle [0.1,0.4], [0,0], [0,0] \rangle & \langle [0,0], [0,0], [0,0] \rangle \end{bmatrix}$$

3 An Application of Interval-valued Neutrosophic Soft Graphs in a Decision Making Problem

Interval-valued neutrosophic soft set has various applications in decision making problems and used to deal with uncertainties from our real life problems. In this section, we applied the concept of interval-valued neutrosophic soft sets in a decision making problem and then gave an algorithm for the selection of optimal object based on given sets of information. Suppose that $V = \{t_1, t_2, t_3, t_4, t_5\}$ be the set of five football clubs under consideration which an investor is going to purchase a football club on the basis of wishing parameters or attributes set $A = \{e_1 = \text{performance}, e_2 = \text{status}, e_3 = \text{squad value}\}$.

Let (K, A) be an interval-valued neutrosophic soft graphs on V which describe the value of the football clubs based upon the given parameters $e_1 = \text{performance}$, $e_2 = \text{status}$, $e_3 = \text{squad value}$, respectively, as given by

$$\begin{aligned} K(e_1) &= \{ \langle t_1, [0.1, 0.2], [0.5, 0.6], [0.3, 0.5] \rangle, \langle t_2, [0.2, 0.5], [0.4, 0.7], [0.5, 0.8] \rangle, \\ &\quad \langle t_3, [0.5, 0.6], [0.6, 0.8], [0.1, 0.3] \rangle, \langle t_4, [0.3, 0.4], [0.2, 0.3], [0.1, 0.3] \rangle, \\ &\quad \langle t_5, [0.5, 0.6], [0.7, 0.8], [0.1, 0.3] \rangle \} \\ K(e_2) &= \{ \langle t_1, [0.3, 0.4], [0.4, 0.7], [0.1, 0.2] \rangle, \langle t_2, [0.3, 0.5], [0.2, 0.3], [0.1, 0.2] \rangle, \\ &\quad \langle t_3, [0.7, 0.8], [0.5, 0.6], [0.4, 0.5] \rangle, \langle t_4, [0.2, 0.4], [0.1, 0.2], [0.8, 0.9] \rangle, \\ &\quad \langle t_5, [0.4, 0.5], [0.3, 0.4], [0.6, 0.7] \rangle \} \\ K(e_3) &= \{ \langle t_1, [0.2, 0.3], [0.3, 0.5], [0.2, 0.4] \rangle, \langle t_2, [0.4, 0.5], [0.3, 0.4], [0.5, 0.7] \rangle, \\ &\quad \langle t_3, [0.6, 0.7], [0.4, 0.5], [0.3, 0.4] \rangle, \langle t_4, [0.3, 0.4], [0.2, 0.3], [0.6, 0.7] \rangle, \\ &\quad \langle t_5, [0.1, 0.2], [0.4, 0.5], [0.5, 0.6] \rangle \}. \end{aligned}$$

Let (M, A) be an interval-valued neutrosophic soft set on $E = \{t_1t_2, t_1t_3, t_1t_4, t_1t_5, t_2t_3, t_2t_4, t_2t_5, t_3t_4, t_3t_5, t_4t_5\}$ which describe the value obtained by comparing two football clubs according to each parameter as given by

$$\begin{aligned}
 M(e_1) &= \{ \langle t_1t_2, [0.1, 0.2], [0.6, 0.8], [0.6, 0.9] \rangle, \langle t_2t_3, [0.1, 0.4], [0.8, 0.9], [0.6, 0.9] \rangle, \\
 &\quad \langle t_1t_5, [0.1, 0.2], [0.8, 0.9], [0.4, 0.6] \rangle, \langle t_2t_4, [0.2, 0.3], [0.7, 0.8], [0.7, 0.9] \rangle, \\
 &\quad \langle t_3t_4, [0.2, 0.3], [0.7, 0.8], [0.2, 0.4] \rangle, \langle t_2t_5, [0.1, 0.3], [0.8, 0.9], [0.5, 0.8] \rangle, \\
 &\quad \langle t_3t_5, [0.3, 0.4], [0.7, 0.9], [0.5, 0.6] \rangle \}, \\
 M(e_2) &= \{ \langle t_1t_2, [0.2, 0.3], [0.5, 0.6], [0.3, 0.4] \rangle, \langle t_2t_3, [0.3, 0.4], [0.6, 0.7], [0.5, 0.6] \rangle, \\
 &\quad \langle t_1t_5, [0.1, 0.2], [0.6, 0.8], [0.8, 0.9] \rangle, \langle t_3t_4, [0.2, 0.3], [0.6, 0.7], [0.8, 0.9] \rangle, \\
 &\quad \langle t_2t_5, [0.3, 0.4], [0.4, 0.5], [0.7, 0.8] \rangle \}, \\
 M(e_3) &= \{ \langle t_1t_2, [0.1, 0.2], [0.4, 0.6], [0.6, 0.8] \rangle, \langle t_2t_3, [0.3, 0.4], [0.5, 0.6], [0.7, 0.8] \rangle, \\
 &\quad \langle t_1t_5, [0.1, 0.2], [0.5, 0.6], [0.6, 0.7] \rangle, \langle t_3t_4, [0.1, 0.2], [0.6, 0.7], [0.6, 0.8] \rangle, \\
 &\quad \langle t_4t_5, [0.1, 0.2], [0.5, 0.6], [0.7, 0.8] \rangle, \langle t_2t_4, [0.2, 0.3], [0.4, 0.5], [0.8, 0.9] \rangle, \\
 &\quad \langle t_2t_5, [0.1, 0.2], [0.7, 0.8], [0.8, 0.9] \rangle \}.
 \end{aligned}$$

The interval-valued neutrosophic graphs $H(e_1) = (K(e_1), M(e_1))$, $H(e_2) = (K(e_2), M(e_2))$ and $H(e_3) = (K(e_3), M(e_3))$ corresponding to the parameters e_1 , e_2 and e_3 , respectively, are represented by the following incidence matrices.

$$\begin{aligned}
 H(e_1) &= \begin{bmatrix} \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.6, 0.8], [0.6, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.8, 0.9], [0.4, 0.6] \rangle \\ \langle [0.1, 0.2], [0.6, 0.8], [0.6, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.4], [0.8, 0.9], [0.6, 0.9] \rangle & \langle [0.2, 0.3], [0.7, 0.8], [0.7, 0.9] \rangle & \langle [0.1, 0.3], [0.8, 0.9], [0.5, 0.8] \rangle \\ \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.4], [0.8, 0.9], [0.6, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.2, 0.3], [0.7, 0.8], [0.2, 0.4] \rangle & \langle [0.3, 0.4], [0.7, 0.9], [0.5, 0.6] \rangle \\ \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.2, 0.3], [0.7, 0.8], [0.7, 0.9] \rangle & \langle [0.2, 0.3], [0.7, 0.8], [0.2, 0.4] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \\ \langle [0.1, 0.2], [0.8, 0.9], [0.4, 0.6] \rangle & \langle [0.1, 0.3], [0.8, 0.9], [0.5, 0.8] \rangle & \langle [0.3, 0.4], [0.7, 0.9], [0.5, 0.6] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \end{bmatrix} \\
 H(e_2) &= \begin{bmatrix} \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.2, 0.3], [0.5, 0.6], [0.3, 0.4] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.6, 0.8], [0.8, 0.9] \rangle \\ \langle [0.1, 0.2], [0.6, 0.8], [0.8, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.3, 0.4], [0.6, 0.7], [0.5, 0.6] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.3, 0.4], [0.4, 0.5], [0.7, 0.8] \rangle \\ \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.3, 0.4], [0.6, 0.7], [0.5, 0.6] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.2, 0.3], [0.6, 0.7], [0.8, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \\ \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.2, 0.3], [0.6, 0.7], [0.8, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \\ \langle [0.1, 0.2], [0.6, 0.8], [0.8, 0.9] \rangle & \langle [0.3, 0.4], [0.4, 0.5], [0.7, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \end{bmatrix} \\
 H(e_3) &= \begin{bmatrix} \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.4, 0.6], [0.6, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.5, 0.6], [0.6, 0.7] \rangle \\ \langle [0.1, 0.2], [0.4, 0.6], [0.6, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.3, 0.4], [0.5, 0.6], [0.7, 0.8] \rangle & \langle [0.2, 0.3], [0.4, 0.5], [0.8, 0.9] \rangle & \langle [0.1, 0.2], [0.7, 0.8], [0.8, 0.9] \rangle \\ \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.3, 0.4], [0.5, 0.6], [0.7, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.6, 0.7], [0.6, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \\ \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.2, 0.3], [0.4, 0.5], [0.8, 0.9] \rangle & \langle [0.1, 0.2], [0.6, 0.7], [0.6, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.5, 0.6], [0.7, 0.8] \rangle \\ \langle [0.1, 0.2], [0.5, 0.6], [0.6, 0.7] \rangle & \langle [0.1, 0.2], [0.7, 0.8], [0.8, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.5, 0.6], [0.7, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \end{bmatrix}
 \end{aligned}$$

After performing parametric \vee and parametric \wedge operations we obtain two resultant interval-valued neutrosophic graph $H(e)$ and $H'(e)$. The incidence matrices of $H(e)$ and $H'(e)$ are given by as follows.

$$H(e) = \begin{bmatrix} \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.4, 0.6], [0.6, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.5, 0.6], [0.6, 0.7] \rangle \\ \langle [0.1, 0.2], [0.4, 0.6], [0.6, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.3, 0.4], [0.5, 0.6], [0.7, 0.8] \rangle & \langle [0.2, 0.3], [0.4, 0.5], [0.8, 0.9] \rangle & \langle [0.1, 0.2], [0.7, 0.8], [0.8, 0.9] \rangle \\ \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.3, 0.4], [0.5, 0.6], [0.7, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.6, 0.7], [0.6, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \\ \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.2, 0.3], [0.4, 0.5], [0.8, 0.9] \rangle & \langle [0.1, 0.2], [0.6, 0.7], [0.6, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.5, 0.6], [0.7, 0.8] \rangle \\ \langle [0.1, 0.2], [0.5, 0.6], [0.6, 0.7] \rangle & \langle [0.1, 0.2], [0.7, 0.8], [0.8, 0.9] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle & \langle [0.1, 0.2], [0.5, 0.6], [0.7, 0.8] \rangle & \langle [0, 0], [0, 0], [0, 0] \rangle \end{bmatrix}$$

$$H'(e) = \begin{bmatrix} \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.2], [0.6,0.8], [0.6,0.9] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.2], [0.8,0.9], [0.8,0.9] \rangle \\ \langle [0.1,0.2], [0.6,0.8], [0.6,0.9] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.4], [0.8,0.9], [0.7,0.9] \rangle & \langle [0,0], [0.7,0.8], [0.8,0.9] \rangle & \langle [0.1,0.2], [0.8,0.9], [0.8,0.9] \rangle \\ \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.4], [0.8,0.9], [0.7,0.9] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0.1,0.2], [0.7,0.8], [0.8,0.9] \rangle & \langle [0,0], [0.7,0.9], [0.5,0.6] \rangle \\ \langle [0,0], [0,0], [0,0] \rangle & \langle [0,0], [0.7,0.8], [0.8,0.9] \rangle & \langle [0.1,0.2], [0.7,0.8], [0.8,0.9] \rangle & \langle [0,0], [0,0], [0,0] \rangle & \langle [0,0], [0.5,0.6], [0.7,0.8] \rangle \\ \langle [0.1,0.2], [0.8,0.9], [0.8,0.9] \rangle & \langle [0.1,0.2], [0.8,0.9], [0.8,0.9] \rangle & \langle [0,0], [0.7,0.9], [0.5,0.6] \rangle & \langle [0,0], [0.5,0.6], [0.7,0.8] \rangle & \langle [0,0], [0,0], [0,0] \rangle \end{bmatrix}$$

The average possible membership degree of an element x in given interval-valued neutrosophic set

$$A = \{ \langle x, [infT_A(x), supT_A(x)], [infI_A(x), supI_A(x)], [infF_A(x), supF_A(x)] \rangle \}$$

is calculated by

$$\begin{aligned} S(x) &= \frac{1}{3} \left[\frac{infT_A(x) + supT_A(x)}{2} + 1 - \frac{infI_A(x) + supI_A(x)}{2} + 1 - \frac{infF_A(x) + supF_A(x)}{2} \right] \\ &= \frac{infT_A(x) + supT_A(x) + 4 - infI_A(x) - supI_A(x) - infF_A(x) - supF_A(x)}{6} \end{aligned}$$

Based on S_k we construct the tabular representation of score value of incidence matrix of resultant interval-valued neutrosophic graphs $H(e)$ and $H'(e)$ with S_k and choice value for each football club t_k for $k = 1, 2, 3, 4, 5$

Table 2: Tabular representation of score values with choice values of $H(e)$.

	t_1	t_2	t_3	t_4	t_5	t'_k
t_1	0.667	0.317	0.667	0.667	0.367	2.683
t_2	0.317	0.667	0.417	0.750	0.417	2.567
t_3	0.667	0.417	0.667	0.433	0.783	2.967
t_4	0.667	0.750	0.433	0.667	0.717	3.233
t_5	0.367	0.417	0.783	0.717	0.667	2.950

Table 3: Tabular representation of score values with choice values of $H'(e)$

	t_1	t_2	t_3	t_4	t_5	t''_k
t_1	0.667	0.233	0.667	0.667	0.150	2.383
t_2	0.233	0.667	0.200	0.133	0.150	1.383
t_3	0.667	0.200	0.667	0.183	0.217	1.933
t_4	0.667	0.133	0.183	0.667	0.233	1.883
t_5	0.150	0.150	0.217	0.667	0.667	1.417

After taking the arithmetic average of t'_k and t''_k for all $k \in \Lambda = \{1, 2, 3, 4, 5\}$, we obtain the table shown in the below.

Table 4: Tabular representation of average score values of $H(e)$ and $H'(e)$.

	t'_k	t''_k	t_k
t_1	2.383	2.683	2.533
t_2	1.383	2.567	1.975
t_3	1.933	2.967	2.450
t_4	1.833	3.233	2.558
t_5	1.417	2.950	2.183

Clearly, the maximum score value is 2.558, scored by the t_4 . Then the investor should purchase the football club t_4 which is the best choice for him.

Now we present our approach as an algorithm that is used in our application to deal with a decision-making problem.

Algorithm

- (i) Input the set P of choice of parameters of an investor or a businessman, A is subset of P .
- (ii) Input the interval-valued neutrosophic soft sets (K, A) and (M, A) .
- (iii) Construct the interval-valued neutrosophic soft graph $\tilde{G} = (G^*, K, M, A)$.
- (iv) Compute the resultant interval-valued neutrosophic graphs $H(e)$ and $H'(e)$ with

$$H(e) = \bigvee_{k \in \Lambda} H(e_k) \text{ for } e = \bigvee_{k \in \Lambda} e_k, \text{ for all } k \in \Lambda.$$

$$H'(e) = \bigwedge_{k \in \Lambda} H(e_k) \text{ for } e = \bigwedge_{k \in \Lambda} e_k, \text{ for all } k \in \Lambda.$$

- (v) Consider the interval-valued neutrosophic graph $H(e)$ and $H'(e)$, and construct their incidence matrix forms.
- (vi) Calculate the score S_k of t_k for all $k \in \Lambda$.
- (vii) The decision is t_k if $t'_k = \max t_k$.

4 Conclusion

Graph theory is an extremely useful mathematical tool to solve the complicated problems in different fields. The interval-valued neutrosophic soft models give more sensitive, flexibility and conformity to the systems as compared to the interval-valued fuzzy soft models. We have applied the notion of interval-valued neutrosophic soft graph in a decision making problem. Also, a case study has been taken to exhibit the simplicity of the technique. Future work in this regard would be required to study whether the notions put forward in this paper yield a fruitful result.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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